

CERN-PH-TH/2011-027

Three Point Tree Level Amplitude in Superstring Theory

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In order to check the definite amplitude and the exact zero result of the amplitude of three massless points (CAA) in both string theory and field theory side for $p = n$ case and to find all gauge field couplings to R-R closed string, we investigate the disk level S-matrix element of one Ramond-Ramond field and two gauge field vertex operators in the world volume of BPS branes.

1. Introduction

Due to the fact that D_p -branes are the source of Ramond-Ramond $(p+1)$ -form fields in IIA and IIB string theories [1], and that many properties of them have been investigated [2,3], they are of high importance in both theory and phenomenology. The stable D_p -branes (p is even in IIA and odd in IIB theory) preserve half of supersymmetry.

Stability, supersymmetry, conserved Ramond-Ramond (RR) charge and having no tachyons are, in fact, all properties of these type II D_p -branes. All supersymmetric D_p -branes in IIA can be generated as bound states of D_9 -branes [4]. They can also be derived from K-theory [5]. At leading order, the low-energy action for fields corresponds to dimensional reduction of a ten-dimensional $U(1)$ super-Yang Mills theory. When derivatives of the field strengths are small on the string scale, the action to all orders in the field strength takes the Born-Infeld form [6,7] (also see [8]). The low energy action describing the dynamics of D_p -branes consists of two parts. The first part is the Born-Infeld action (for more details see [9]). In addition to providing the kinetic terms for the world-volume fields, the BI action contains the couplings of the D_p -brane to the massless Neveu-Schwarz fields in the bulk. The second part is the Wess-Zumino action, which contains the coupling of the $U(N)$ massless world volume vectors to the closed string RR field [1,10]. One method for finding these effective actions is BSFT [11]. To study WZ couplings for BPS branes, we

use the S-matrix method as the second approach.

2. The Three Point Superstring Scattering (CAA)

One important tool in string theory is scattering theory [12]. In this section, using the conformal field theory techniques [13], we evaluate this scattering amplitude to find all couplings of one closed string RR field to two gauge fields on the world-volume of a single BPS D_p -brane with flat empty space background. A great deal of effort for the understanding of scattering amplitudes at tree level has been made [14]. Some previous works on scattering including a D_p -brane and some other works about applications on D_p -brane can be found in [15].

To calculate a S-matrix element, one needs to choose the picture of the vertex operators appropriately. The sum of the superghost charge must be -2 for the disk level amplitude. Hence, this S-matrix element is given by the following correlation function

$$\mathcal{A}^{AAC} \sim \int dx_1 dx_2 dz d\bar{z} \langle V_A^{(-1)}(x_1) V_A^{(0)}(x_2) \times V_{RR}^{(-1/2, -1/2)}(z, \bar{z}) \rangle, \quad (1)$$

where the vertex operator and the “doubling trick” have been mentioned in [16]. The Wick-like rule [17] is used to find the correlator of ψ . The only subtlety in the use of this formula for currents is that the Wick-like contraction for the two ψ 's in one current [18] should not be consid-

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ered, so that it follows,

$$\mathcal{A}^{CAA} \sim \int dx_1 dx_2 dx_3 dx_4 (P_- \mathbb{H}_{(n)} M_p)^{\alpha\beta} I \xi_{1a} \\ \times \xi_{2b} x_{34}^{-1/4} (x_{23} x_{24})^{-1/2} (a_1^a a_2^b + 2i k_{1c} I^{bac}),$$

where

$$I = |x_{12}|^{-2s} |x_{13} x_{14}|^s |x_{23} x_{24}|^s |x_{34}|^{-2s} \\ a_1^a = i k_2^a \left(\frac{x_{32}}{x_{13} x_{12}} + \frac{x_{42}}{x_{14} x_{12}} \right) \\ a_2^b = 2^{-1/2} x_{34}^{-3/4} (x_{23} x_{24})^{-1/2} (\gamma^b C^{-1})_{\alpha\beta}.$$

For details about I^{bac} see [16]. We define the Mandelstam variable as $s = -\frac{\alpha'}{2}(k_1 + k_2)^2$. Clearly it is seen that the integrand is invariant under $SL(2, \mathbb{R})$ transformation. Gauge fixing this symmetry by fixing the position of the closed string vertex operator as

$$x_1 = x, \quad x_2 = -x, \quad x_3 = i, \quad x_4 = -i,$$

the amplitude becomes :

$$\mathcal{A} = \mathcal{A}_1 + \mathcal{A}_2 + \mathcal{A}_3, \quad (2)$$

where

$$\mathcal{A}_1 \sim i k_2^a (2i)^{-2s} 2^{-1/2} \xi_{1a} \xi_{2b} \text{Tr} (P_- \mathbb{H}_{(n)} M_p \gamma^b) \\ \times \int_0^\infty dx \frac{1}{x} (x^2 - 1) (2x)^{-2s} (x^2 + 1)^{2s-1} \\ \mathcal{A}_2 \sim -k_{1c} (2i)^{-2s} 2^{1/2} \xi_{1a} \xi_{2b} \text{Tr} (P_- \mathbb{H}_{(n)} M_p \Gamma^{bac}) \\ \times \int_0^\infty dx (2x)^{-2s} (x^2 + 1)^{2s-1} \quad (3) \\ \mathcal{A}_3 \sim -k_{1c} (2i)^{-2s} 2^{1/2} \xi_{1a} \xi_{2b} (\eta^{cb} \text{Tr} (P_- \mathbb{H}_{(n)} M_p \gamma^a) \\ - \eta^{ab} \text{Tr} (P_- \mathbb{H}_{(n)} M_p \gamma^c)) \\ \times \int_0^\infty dx \frac{1}{2ix} (-x^2 + 1) (2x)^{-2s} (x^2 + 1)^{2s-1}.$$

To obtain the definite amplitude, the real part of s has to be less than zero. In addition all integrations must be taken over the positive values of x , otherwise the terms x^{-s} will give rise to a complex amplitude. Thus, the only non vanishing integral is the second one for which the result is

$$\int_0^\infty dx (2x)^{-2s} (x^2 + 1)^{2s-1} = \frac{\pi^{1/2} \Gamma[-s + 1/2]}{2\Gamma[-s + 1]}.$$

To compare the field theory which apparently has massless field, *i.e.*, WZ action, with the above amplitude, it must be expanded such that the massless pole of the field theory survives and all other poles disappear in the form of contact terms. Note that the S-matrix element of all four point massless vertex operators in superstring theory is also found in standard books [19,20].

3. Momentum expansion

Our goal is to examine the limit of $\alpha' \rightarrow 0$ of the above string amplitude. Applying momentum conservation along the world volume of the brane, we see that the Mandelstam variable satisfies the constraint

$$s = -p_a p^a / 2. \quad (4)$$

It has been shown in [16] that the momentum expansion of a S-matrix element should be in general around $(k_i + k_j)^2 \rightarrow 0$ and/or $k_i \cdot k_j \rightarrow 0$. The amplitude must only have a massless pole in the $(k_1 + k_2)^2$ channel, so that the correct momentum expansion at the low energy limit for s-channel must be around $(k_1 + k_2)^2 \rightarrow 0$. Using the on-shell relations, they can be rewritten in terms of the Mandelstam variable as $s \rightarrow 0$. Under the constraint (4), note that $p_a p^a \rightarrow 0$ is allowed for D-branes. Therefore the S-matrix element can be evaluated for BPS branes. Thus, expansion of the functions around the above point will be

$$(2)^{-2s} \frac{\pi^{1/2} \Gamma[-s + 1/2]}{\Gamma[-s + 1]} = \pi \left(\sum_{n=-1}^\infty b_n(s)^{n+1} \right).$$

where some of the coefficients b_n are

$$b_{-1} = 1, \quad b_0 = 0, \quad b_1 = \frac{1}{6} \pi^2, \quad b_2 = 2\zeta(3).$$

As expected, it is seen that the obtained coefficients b_n and the coefficients appearing in the momentum expansion of the S-matrix element of one RR, two gauge fields and one tachyon vertex operator [21] are exactly the same.

4. Low Energy Field Theory

We focused on the part of effective field theory of D-branes which includes only gauge fields.

It is possible to extract the necessary terms from the covariant Born-Infeld action constructed as the effective D-brane action. The Born-Infeld action is an action for all orders of α' . As low energy non-abelian extension of the action, the symmetrized trace of non-abelian generalization of Born-Infeld action was proposed. The non-abelian field strength and covariant derivative of the gauge field are defined, respectively, as

$$F^{ab} = \partial^a A^b - \partial^b A^a - i[A^a, A^b], \quad (5)$$

$$D_a F_{bc} = \partial_a F_{bc} - i[A_a, F_{bc}].$$

where $A_a = A_a^\alpha \Lambda_\alpha$ and Λ_α are the hermitian matrices. Our conventions for Λ^α are

$$\sum_\alpha \Lambda_{ij}^\alpha \Lambda_{kl}^\alpha = \delta_{ik} \delta_{jl}, \quad \text{Tr}(\Lambda^\alpha \Lambda^\beta) = \delta^{\alpha\beta}.$$

5. $p = n + 2$ case

Due to the fact that only \mathcal{A}_2 is non-zero, we are not interested in fixing the overall sign of the amplitudes. Taking into account the related trace thus the string amplitude should be

$$\mathcal{A}^{CAA} = \pm(\mu_p \pi^{1/2}) \frac{32}{(p-2)!} k_{1c} \xi_{1a} \xi_{2b} \epsilon^{baca_1 \dots a_{p-3}} \\ \times H_{a_1 \dots a_{p-3}} (2)^{-2s} \frac{\pi^{1/2} \Gamma[-s+1/2]}{2\Gamma[-s+1]},$$

where we normalized the amplitude by $(\mu_p 2^{1/2} \pi^{1/2})$. This amplitude is zero upon interchanging gauge fields, rendering the whole amplitude is zero for an abelian gauge group. The amplitude also satisfies the Ward identity. Since the Gamma function has no tachyon/massless pole, the amplitude only has contact terms. The leading contact term is reproduced by the coupling

$$\frac{1}{2!} \mu_p (2\pi\alpha')^2 \text{Tr}(C_{p-3} \wedge F \wedge F). \quad (6)$$

The non-leading order terms have to correspond to the higher derivative extension of the above coupling. Thus, the higher vertex will be

$$V(C_{p-3}, A_3, A) = \frac{\mu_p (2\pi\alpha')^2}{(p-2)!} \epsilon^{a_1 \dots a_{p+1}} H_{a_1 \dots a_{p-2}} \\ \times \xi_{1a_{p-1}} k_{1a_p} \xi_{2a_{p+1}} \sum_{n=-1}^{\infty} b_n (\alpha' k_1 \cdot k_2)^{n+1}.$$

6. $p = n$ case

Despite the fact that in string theory both $\mathcal{A}_1, \mathcal{A}_3$ are zero in this case, we would like to perform the field theory calculations to confirm that, there is no compensation of the massless pole,

$$\mathcal{A} = V_\alpha^a(C_{p-1}, A) G_{\alpha\beta}^{ab}(A) V_\beta^b(A, A_1, A_2),$$

where the vertices and propagator are

$$V_\alpha^a(C_{p-1}, A) = \frac{i\mu_p (2\pi\alpha')}{(p)!} \epsilon^{a_0 \dots a_{p-1} a} \\ \times H_{a_0 \dots a_{p-1}} \text{Tr}(\Lambda_\alpha),$$

$$V_\beta^b(A, A_1, A_2) = \left[\xi_1^b(k_1 - k) \cdot \xi_2 + \xi_2^b(k - k_2) \cdot \xi_1 \right. \\ \left. + \xi_1 \cdot \xi_2 (k_2 - k_1)^b \right] (-iT_p (2\pi\alpha')^2 \text{Tr}(\lambda_1 \lambda_2 \Lambda_\beta)),$$

$$G_{\alpha\beta}^{ab}(A) = \frac{i\delta_{\alpha\beta} \delta^{ab}}{(2\pi\alpha')^2 T_p(s)}.$$

α, β and a, b are the group and world volume indices, respectively. The propagator is derived from the standard gauge kinetic term arising in the expansion of the Born-Infeld action. Note that the vertex $V_\beta^b(A, A_1, A_2)$ is found from the standard non abelian kinetic term of the gauge field. Also, the vertex $V_\alpha^a(C_{p-1}, A)$ is found from the WZ coupling $C_{p-1} \wedge F$. In the above formula k is the momentum of the off-shell gauge field. The important point should be made is that the vertex $V_\beta^b(A, A_1, A_2)$ has no higher derivative correction as it arises from the kinetic term of the gauge field. This vertex has already been found in [16]. Considering those vertices, the amplitude yields

$$\mathcal{A} = [\xi_{1a}(k_1 - k) \cdot \xi_2 + \xi_{2a}(k - k_2) \cdot \xi_1 + \xi_1 \cdot \xi_2 \\ (k_2 - k_1)_a] i\mu_p (2\pi\alpha') \frac{1}{(p)! s} \text{Tr}(\lambda_1 \lambda_2) \\ \times \epsilon^{a_0 \dots a_{p-1} a} H_{a_0 \dots a_{p-1}}, \quad (7)$$

which of course describes the apparent massless pole in field theory, while there is no massless pole in string theory.

7. Remarks

In the $p = n$ case, there is no massless pole at $s = 0$. It can be concluded that the kinematic factor provides a compensating factor of s , but we do not know how the compensation is achieved.

To understand the vanishing amplitude, remember that to produce the correct amplitude, we must consider all possible orderings of non abelian gauge fields, which means that we must consider resulting terms by interchanging 1 to 2 in (7) as well. Note that the quantity in the square bracket in (7) is antisymmetric under interchanging 1 to 2. Therefore, apart from the coefficients, the final result for the amplitude is given by

$$\begin{aligned} \mathcal{A} = & [\xi_{1a}(k_1 - k) \cdot \xi_2 + \xi_{2a}(k - k_2) \cdot \xi_1 + \xi_1 \cdot \xi_2 \\ & \times (k_2 - k_1)_a] \epsilon^{a_0 \dots a_{p-1} a} H_{a_0 \dots a_{p-1}} \\ & \times (\text{Tr}(\lambda_1 \lambda_2) - \text{Tr}(\lambda_2 \lambda_1)). \end{aligned}$$

The fact that the amplitude is zero, indicates two concrete points. First, gauge fixing has to be done over the positive values of x ; otherwise we will have a complex amplitude. Therefore, the upper and lower bound of the integration in string theory have been chosen correctly. Second, there was an apparent massless pole in field theory. However, the amplitude vanishes not because of compensating Mandelstam variable, but because of considering all orderings of gauge fields. There is no contact term for $p = n$ case.

Acknowledgments

The author would like to thank G.Veneziano for very beneficial and enjoyable collaboration throughout the project. He would also like to thank his advisors Luis Álvarez-Gaumé and M.R. Garousi for several valuable discussions. The author also acknowledges N. Arkani-hamed, L.Dixon, I.Antoniadis, C.Grojean, P.Vanhove, M. Douglas, N.Lambert, J.Drummond and A. Strominger for comments, useful suggestions and the CERN Theory Group for its hospitality. This work was supported Under the Marie Curie or the EU grant UNILHC PITN-GA-2009-237920.

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